Probabilistic uncertainty in differential games and control

Marc Quincampoix

Laboratoire de Mathématiques de Bretagne Atlantique (CNRS UMR 6205),
Université de Brest, France

In classical optimal control and in differential games, the controllers are supposed to a have a perfect knowledge of the dynamics, of the payoffs and of the initial conditions of the system. However in several practical situations only partial informations on these data are available.

The most simple example is a control system with a given terminal payoff where the initial condition is not perfectly known: only a probabilistic information is known (for instance, the initial condition lies in a given ball with a uniform probability measure). The initial condition is then replaced by a probability measure which «propagates» according to the control system. One can be interested to characterize the optimal mean value of the cost. We will show that this value function – which depends on the initial probability measure – could be characterized as the unique solution – in a suitable sense – of a Hamilton Jacobi Bellmann equation stated on the space of probability measures. This brings some technical difficulties where the optimal transport theory is an important tool.

Another problem concerns two-player antagonistic differential games with incomplete information. One can consider a game where one player perfectly knows the initial condition while his opponent has a partial information of probabilistic nature on the initial position. During the game, both players perfectly observe the actions of their opponents. Since the uninformed player does not know the current state, he can only try to guess it by observing the actions of his opponent. The informed player's interest is also not to reveal too much information to the other player during the game, so he can try to «hide» the missing information by using random strategies. The existence of a value – in mixed strategies – is a crucial question which can be solved by showing that the value exists because it is the unique solution of some Hamilton Jacobi Isaacs equation.

Control on the space of probability measure can also be used for modelizing systems with a large number of agents, viewed as controllers, where all the individual agents have the same microscopic dynamics which may also depend on the macroscopic dynamic of the entire crowd (viewed as statistic hence a probability measure). Relevant control problems may concern the optimal behavior to the crowd of all agents with respect to a prescribed payoff. Once again the dynamics stated on the space of probability measures can be used in this framework.

References

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