Infinite-horizon optimal control.
Some recent advances and applications in economic growth theory

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Infinite-horizon optimal control problems naturally arise in studying different models of optimal dynamic allocation of economic resources, in particular, in growth theory. Typically, the initial state is fixed and the terminal state (at infinity) is free in such problems, while the utility functional to be maximized is given by an improper integral on the time interval $[0,\infty)$. Although the state at infinity is not constrained the maximum principle for such problems may not hold in the normal form, and the standard transversality conditions at infinity may fail. Additional difficulties arise when the model involves a natural resource (renewable or not renewable) as an essential factor of production. In this case, typically, admissible controls are only bounded in an integral sense, which precludes the direct application of the standard existence results. The talk is devoted to some recent results in this field of optimal control and their applications in growth theory.

First, we discuss a special version of the Pontryagin maximum principle for infinite-horizon optimal control problems that was developed in collaboration with V.M. Veliov. The main feature of this result consists in the fact that the adjoint variable is specified explicitly by a formula which is similar to the Cauchy formula for solutions to linear differential systems. Such a description of the adjoint variable provides a convenient analytical tool for studying many problems of optimal economic growth. In a number of situations, the description can be used to validate the application of the standard transversality conditions at infinity and, if these condition are violated, the representation of the adjoint variable by means of the Cauchy-type formula can be used as an alternative for them. Moreover, in optimal economic growth problems, this formula makes it possible to give economic meaning to the adjoint variable of the maximum principle without any a priori assumptions on the smoothness of the optimal value function.

Second, we consider a recently developed existence theorem for the optimal control in an infinite-horizon problem in a situation when the set of geometric constraints on the control is not necessarily bounded. Such problems with unbounded controls arise in many economic applications, for example, in the study of optimal economic growth models that involve renewable or nonrenewable natural resources, when the resource exploitation rate can be arbitrarily large and the constraints imposed on the admissible control have integral form. The proof of this existence result employs the Cauchy-type formula for the adjoint variable of the maximum principle.

Third, as a meaningful example, a model of optimal economic growth with a renewable resource that was recently developed in collaboration with T. Manzoor is presented. The model is formulated as an infinite-horizon optimal control problem with logarithmic instantaneous utility. The problem involves unbounded controls and the non-concave Hamiltonian. These preclude direct application of the standard existence results and Arrow's sufficient conditions for optimality. The study of the model is based on the developed existence result and application of the appropriate version of the Pontryagin maximum principle for infinite-horizon problems.

References

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